



# *At least*, numerals and the QUD

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# *At least* and numerals

- Widespread in news reports etc.
  - “At least  $n$  people killed/injured/made homeless/made redundant”
  - Occasional non-numerical complements (“at least April”)
- Why?
  - That is, why “at least” rather than some other means of expression?

# Redundancy of superlative quantifiers?

- Geurts and Nouwen (2007): on traditional semantics, don't need both superlative and comparative quantifiers
  - *at least*  $n \Leftrightarrow$  *more than*  $n-1$
  - *at most*  $n \Leftrightarrow$  *fewer/less than*  $n+1$
  - Yet, widely assumed/argued that human language abhors synonymy

# Mathematical objection, in excessive detail

- *at least / more than* not interdefinable on  $\mathbb{R}$  or  $\mathbb{Q}$ 
  - or indeed on any dense set
  - “more than 2 metres”  $\neq$  “at least  $x$  metres” for any  $x$
  - “more than three-sevenths of a percent”  $\neq$  “at least  $x$  of a percent” for any  $x$
- Why?
  - Let  $S$  be the set of values that lie within the range “more than  $n$ ”
  - Let  $T$  be the set of values that lie within the range “at least  $x$ ”
  - If “more than  $n$ ” = “at least  $x$ ”,  $S = T$  for this  $n, x$
  - If  $n = x$ ,  $n \in T$  and  $n \notin S$ , hence  $S \neq T$  (contradiction)
  - If  $n < x$ , let  $q = \frac{1}{2}(n + x)$ : then  $n < q < x$   
Then  $q \in S$  and  $q \notin T$ , hence  $S \neq T$  (contradiction)

# Mathematical meta-objection

- *at least / more than* not interdefinable on  $\mathbb{R}$  or  $\mathbb{Q}$ 
  - or indeed on any dense set
  - “more than 2 metres”  $\neq$  “at least  $x$  metres” for any  $x$
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- However:
  - We can arbitrarily closely approximate a “more than” with an “at least”
  - For instance, the only difference between “more than  $n$ ” and “at least  $n$ ” is whether or not  $n$  is in the set
  - In  $\mathbb{R}$  or  $\mathbb{Q}$ , the probability of the value of interest taking the value  $n$  is zero (e.g. nobody is 1.50m tall)

# Modal meaning for *at least*

- G&N: “at least  $n$ ” expresses
  - certainty about the existence of a set of cardinality  $n$  with the predicated property
  - the possibility that the cardinality of the whole set of entities with the predicated property might be greater than  $n$
  - Seems plausible as a description of the meaning of e.g. “At least 43 people have been injured as a result of an accident...”
- However, the modal possibility reading could arise straightforwardly as an implicature
  - Unwillingness just to say  $n$  indicative either of uncooperativeness or of just this kind of ignorance...

# Possible arguments for the pragmatic story

- Conditional contexts
- Distribution with round vs. non-round numbers
- Uncertainty readings from “more than” etc.

# Conditionals

- Apparently felicitous to condition on *at least*
  - “If at least ten people come, I’ll call it a success”
- In this case, the meaning seems strictly “mathematical”
  - This is not “I’ll call it a success if (i) there is a group of 10 people who come and (ii) it’s possible that there will be more”
  - That would be a more stringent condition, although it’s not in general an absurd one
- Convenient pragmatic account:
  - Enrichment in the antecedent of a conditional would result in more stringent conditions, but the resulting conditional would be informationally weaker (in the absence of conditional perfection)



## Again, why *at least*?

- Same problem of synonymy: if it doesn't add anything, why is it possible to use *at least* in antecedents of conditionals?
  - “If ten people come...” – perhaps problematic, attracts exact reading
  - “If more than nine people come...” – potentially more complex? (I return to this later)

# Round vs. non-round complements

- Curious usage patterns if we compare “more than” and “at least” around round numbers (e.g. via Google search)
  - “more than 20 people” > “at least 21 people” (18.4M, 4.3M)
  - “at least 20 people” > “more than 19 people” (8.0M, 85k)
  - Twice as likely to say “at least 20” than “at least 21” here: suggests preference for roundness, but also competition from “more than”
  - Might be surprising if there are profound differences in the meaning intended (how much can the tail wag the dog?)

# “More than” uncertainty?

- (Trying not to pre-empt Mayr & Meyer)
- Speaker of “more than  $n-1$ ” still committed to the existence of a set of cardinality  $n$
- Also has potential to give rise to an uncertainty reading (why not say the precise value?)
  - Speaker of “at least  $n$ ” can reasonably be inferred to be uncertain, but speaker of “more than  $n-1$ ” can’t be inferred to be certain...
  - Is there really a crisp categorical distinction, or is it just a matter of inference strength, cue validity, or whatever?

# Why *at least*, per se?

- *At least* may give rise to implicatures, but these are (by definition) as a consequence of it not being other things
- But what **is** it? What do we mean to achieve by using *at least*?

# Licensing specific construals

- “I will invite at least three people, namely X, Y and Z”
  - Infelicitous with “more than two”
  - OK with “at most three”, not “fewer than four”
  - Generally OK with Nouwen’s (2010) class B modifiers
- Putatively (G&N) difference due to argument structure of comparative modifiers being more restrictive
  - Comparative quantifiers require arguments to be first-order predicates, where superlatives do not
  - Indefinite “three people” can acquire a specific construal in the superlative sentence

# Alternative route

- Difference between “at least” and “more than”, on the traditional analysis:
  - “at least  $n$ ” defines a range of values that includes  $n$
  - “more than  $n$ ” defines a range of values that excludes  $n$
- Perhaps the referent “three people” is more accessible because it’s explicitly mentioned by “at least three”
- If so, that could be argued to fall out from the semantic meaning – explicit mention doesn’t happen for “more than”
  - Could the general difference in argument structure be reducible to this?

# Consequence of explicit mention (of number)

- Based on distributional patterns, it looks like the number is identified as a particularly salient reference point
  - “at least 20” vs. “more than 19” – 20 is more comprehensible as a reference point (represented on a coarser-grained scale)
- Use of a non-round number seems to convey that it’s important to the discourse purpose:
  - “Will Tiger win more than 18 majors?”
  - “It’s theoretically possible to score more than 36 runs in an over”
  - “It will take a lot more than 768mph to leave the Earth’s gravitational hold”
- Suggests that the QUD here is not “how many” but “how many relative to the reference point”

# Specific QUD – loss of implicature?

- “more than 80”  $\rightarrow$  “not more than 100” (Cummins, Sauerland and Solt 2012)
- However, for “more than 87” (say) this inference is less stable
- This suggests that
  - the hearer of “more than 80” recovers something like a standard quantity implicature, negating the stronger alternative
  - the hearer of “more than 87” infers instead that the number mentioned is in some sense under discussion
- “More than  $n$ ” with round  $n$  can still be used as an approximation, but with non-round  $n$  this is trickier



# “In fact” class of examples

- “John has more than two children; in fact, he has five”
  - Generally judged to be felicitous
  - Indicates that the speaker of “more than two” can be knowledgeable about the precise value ( $> 3$ )
  - However, also seems to require a context in which the specific issue of “whether or not John has more than two children” is on the table
  - Even if we don’t know that this is present, we can accommodate it, as in the “more than 18”, etc., examples

# Pilot study: inferring grounds for number use

- Using cardinal expressions of quantity from the BNC
  - “more than one/two/three/four”
  - “more than 60/70/80/90”
  - (“more than 58/77/86/93”)
  - “at least 60/70/80/90”
  - (“at least 58/77/86/93”)
- Two balanced lists, each of 12 items with different numbers
  - All four “more than” + small numbers
  - Two “more than” + round numbers, two “more than” + non-round
  - Two “at least” + round numbers, two “at least” + non-round

# Pilot study: inferring grounds for number use

- For each sentence, participants asked to give judgments on 5-point Likert scales for four independent questions
  - i. whether the utterance licensed a specific quantity implicature (e.g. for “more than 70”, asked “whether not more than 80”)
  - ii. whether the utterance was the most informative possible, from the speaker’s point of view
  - iii. whether the utterance was a convenient approximation
  - iv. whether the specific number used was important for some reason
- Predictions:
  - Negative correlation between (i) and (iv)
  - Round numbers to score higher than others on (iii), lower on (iv)

# Results

- Fielded on MTurk: 17 + 14 participants, pooled here

	(i)	(ii)	(iii)	(iv)
<b>More than</b>				
<b>Round</b>	3.46 (1.30)	3.44 (1.15)	4.08 (1.06)	2.98 (1.09)
<b>Small</b>	2.02 (1.27)	3.43 (1.13)	3.29 (1.20)	3.58 (1.24)
<b>Neither</b>	3.63 (1.12)	3.68 (1.04)	3.29 (1.23)	3.11 (1.27)
<b>At least</b>				
<b>Round</b>	3.37 (1.41)	3.67 (1.04)	3.90 (0.94)	3.10 (1.16)
<b>Neither</b>	3.27 (1.38)	3.87 (1.09)	3.21 (1.33)	3.27 (1.26)

- Small numbers regarded as potentially important
- Strong negative correlation (i)/(iv) across items ( $r = -0.67$ )

# Back to “at least”

- Similarly (if not more so), “at least  $n$ ” should raise the issue of “whether or not  $n$ ”
- Given that  $n$  is a threshold of interest, speaker could alternatively have said “more than  $n$ ” or just “ $n$ ”
- If “more than  $n$ ” is simpler, “at least  $n$ ” should implicate the possibility that exactly  $n$  is the case

# Alternatives

- Could see this as a potential solution to the problem of establishing the alternatives for quantity implicature
  - Outside of the scalar case, it's not clear which informationally stronger alternatives ought to be available for negation
  - Possible that we're only interested in alternatives that would be a good answer to the QUD
  - Speaker of "at least  $n$ " could be taken to signal that " $n$ " and "more than  $n$ " would be good answers to the QUD, and hence can enter into this computation

# Status of the “threshold of interest”

- Still need a more precise account of this notion of “threshold of interest”
  - “At least 43 people were injured in the accident”
  - “If there are at least 11 people at the meeting, we can elect a chair”
  - Why don’t these get competed out of existence by “more than 42” / “more than 10”?
    - Note: for “at least 20” vs. “more than 19”, we can argue that the former uses a round number, but that’s not the case here
  - Idea: number is favoured for prior contextual reasons
    - “Quorum size = 11”
    - Then have to use “at least” in order to both use the numeral of interest and express the intended proposition

# The end of the news

- “At least 43 people...”
  - Could it be that the speaker encoding this information has the numeral “43” active for some prior contextual reason?
  - Plausible if we consider the source of the information (paraphrase or direct experience)
  - What would constitute a good empirical test?



# Another (near) circularity

- My research has tended to focus on production as well as interpretation
- This discussion attempts to pursue G&N's insights “upstream”:
  - Instead of proposing that “at least  $n$ ” evokes sets of cardinality  $n$ , I argue that sets of cardinality  $n$  evoke the use of “at least  $n$ ”
  - Superficially very similar claims (under reasonable assumptions about communication)
  - Differences technical and perhaps relate to the kinds of interfaces that we're talking about...
  - ...or maybe I grossly overestimate the differences...

# Thank you!

- Questions? (Or, better: answers?)